MMSE Linear Multiuser Detection for a DS-CDMA System

ECE 6604

Personal and Mobile Communications

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Spring 2000

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Abstract

Code Division Multiple Access (CDMA) is one of the most flexible multiple access methods suitable for supporting many new services such as speech, video, multimedia applications and so on, which are becoming increasingly important for mobile communications. Besides supporting multiple data rates, future systems will also need to enhance the performance and capacity demands. Direct Sequence Code Division Multiple Access (DS-CDMA) is the most commonly proposed CDMA system for the third generation of wireless mobile systems. A DS-CDMA system using conventional receiver techniques is limited in capacity because the multiple access interference (MAI) and the near-far effect will substantially degrade its performance. These problems, however, can be mitigated by employing multiuser detection techniques, including optimum detectors and suboptimum ones.

Although optimum detectors offer high performance and high capacity, it is very difficult to implement them because of excessive computational complexity, which grows exponentially increased with the number of users. The MMSE linear detector, however, has low complexity and yet provides acceptable performance. This paper presents a literature survey of linear MMSE detection and its implementations both in asynchronous and synchronous cases.

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1. Introduction

Recently, there has been an increasing interest in telecommunications towards new services and more flexible communication systems. This chapter will serve as an introduction to the area of cellular communication systems. The concept of a cellular system is introduced in section 1.1. Section 1.2 presents different approaches of multiple access techniques that are widely used in communication systems. Finally, a brief description about DS-CDMA is given in section 1.3.

1.1 Cellular System Concept

The main idea of a cellular system is to divide the system service areas into smaller areas, called *cells*, which are served by separate base stations. The transmitted power of the base stations will limit the coverage service area. In order to increase the capacity of a system, frequency reuse technique must then be employed to reuse the allocated frequency at the closet possible distance without the interference level exceeding tolerable limits.

Several cells in a system are grouped into *clusters* in which different frequencies are allocated to different base stations. The same frequencies are therefore shared in other clusters. The number of cluster sizes, N, can be obtained by

$$N = i^2 + ij + j^2 \tag{1.1}$$

where i and j are arbitrary integer numbers such that $0 \leq i < j$. It is obvious that the allowable number of cluster sizes are N = 1, 3, 4, 7, 9, etc. The capacity of a system is increased if the size of the cells is reduced, as the allocated frequency band can be reused at shorter distances. Nevertheless, this demands an increase in the number of base stations and a decrease in the base stations' transmitted power. The overview of a cellular mobile network with a cluster size of 4 is illustrated in Figure 1.1. It shows how a service area with many cells is connected via a mobile

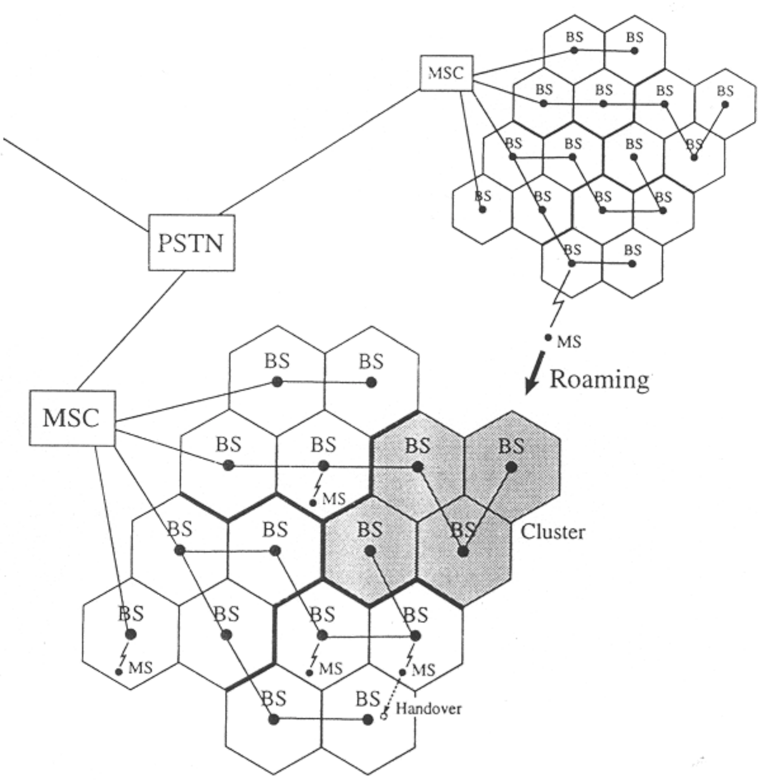

Figure 1.1: The overview of a cellular mobile network with a cluster size of 4 [1].

switching center (MSC) to a public switched telephone network (PSTN). The MSC's main task is to perform functions of a basic digital switching exchange, to determine if a mobile subscriber is available for receiving a call, to monitor the quality of service for subscribers, and to handle handoff, which is the process when a mobile moves from one cell to another such that the service connection is switched over to a new base station without interruption. It is sometimes essential to handoff a call between two cells served by different MSCs, which is commonly occurred when one a mobile moves from one country to another. Such an event is known as "*roaming*".

1.2 Multiple Access Techniques

Within a cluster of cells, multiple access (MA) techniques are employed to allow the users to simultaneously share the available frequency band as efficiently as possible to achieve high capacity. Figure 1.2 illustrates three different MA techniques for allocating resources to the users.

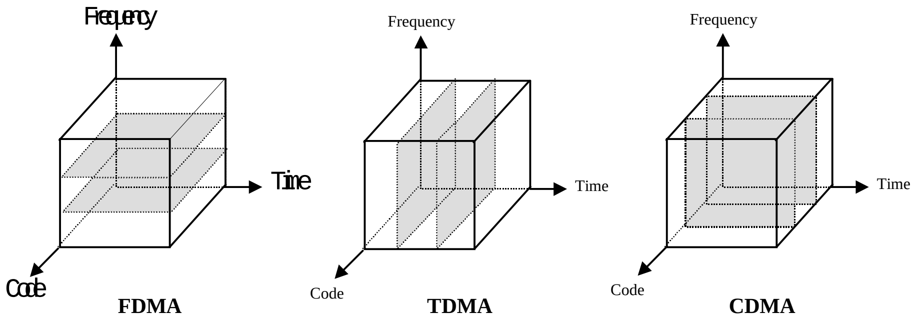

Figure 1.2: Multiple Access Techniques.

1.2.1 Frequency Division Multiple Access (FDMA)

With a FDMA technique, the available bandwidth is divided into frequency bands (or channels) that are assigned to individual users. Hence, each user with his/her occupied frequency band can transmit signals at the same time continuously. Since only one user uses a specific frequency band, its bandwidth is usually narrow and FDMA is therefore classified as a narrowband technique. This method was used in the first analog cellular system such as AMPS in the U.S.

1.2.2 Time Division Multiple Access (TDMA)

TDMA allows each user to communicate with the base station by using the whole bandwidth but different allocated time slots. The transmission and reception of signals occur in different slots separated in time. This makes the transmission in a TDMA system bursty and therefore only suitable for digital modulation techniques because analog modulation techniques generally require continuous transmission. TDMA is usually preferred because of easier implementation and thus less expensive base stations and mobile terminals. The TDMA technique is a dominating access method used in most of the second generation of mobile systems such as the European GSM and the American IS-54.

1.2.2 Code Division Multiple Access (CDMA)

CDMA differs from FDMA and TDMA in the sense that all users can transmit the information signal at the same time using the same frequency band. Each user is assigned a unique spreading code with the purpose of being able to distinguish between different users and their transmissions. The key features of a CDMA system are the continuous transmission and the reduced system administration needed to allocate resources to the users. Furthermore, CDMA is more tolerant to multipath fading than TDMA and FDMA. Although CDMA is more powerful in practice, it is also quite complicated in the process of making synchronization between generating the code sequence of transmitter and decoding coded signal at receiver. Nevertheless, the CDMA technique will be primarily employed for the third generation of wireless personal communication systems such as the American IS-95 and the European W-CDMA.

1.3 Direct Sequence - Code Division Multiple Access (DS-CDMA)

CDMA is known as a spread spectrum technique, since each user is assigned a unique spreading code to spread the narrow band information signal over the whole bandwidth. The most common CDMA methods are frequency-hopping (FH) and direct-sequence (DS). In a FH-CDMA system, the carrier hops from one frequency to another in a pseudo-random hopping-pattern controlled by the spreading code. For a DS-CDMA system, the spreading code is a pseudo-ransom, usually binary sequence, with a much larger bandwidth than the transmitted information signal. The information is multiplied by the spreading code to introduce rapid phase transitions and accordingly increase the signal bandwidth. The DS-CDMA technique is proposed for the third generation of mobile systems.

DS-CDMA has many advantages for employing in the next generation of mobile systems. First, it provides a soft capacity meaning that there is no limit number of users in the system. Second, a frequency reuse factor of one can be deployed, thus allowing the whole spectrum bandwidth to be theoretically reused in every cell. Finally, the soft handoff, referred to the possibility of a mobile station moving close to the cell boundary to establish the new base station, can be implemented. This procedure ensures that the quality of the radio link to the new base station is not disconnected. In addition, in a soft handoff, the mobile station can transmit using less power and thus reducing the interference to intra-cell users as well as to users in neighboring cells. Other advantages of DS-CDMA include the rejection of narrow band interference and the possibility to exploit multipath diversity combining at the receiver.

DS-CDMA, however, experiences two main difficulties, i.e., the near-far problem and the multiple access interference (MAI). In synchronous CDMA systems with no multipath propagation, orthogonal codes can be used and therefore the users do not interfere with each other. However, if the system is not completely synchronized due to multipath propagation, the users will experience MAI. This problem is further accentuated by the near-far problem. The near-far problem occurs especially in the uplink (mobile station to base station) when a weak signal from a distant mobile station is swamped out by the strong signal from a mobile station closer to the base station. Even if the mobile stations are at the same distance from the base station, the channel can introduce fading leading to the same effect.

Stringent power control, where the base station adjusts the power level of the mobile stations so that the power they receive are equal, is one way to combat the near-far problem. If the MAI is kept within reasonable limits by a good code design and a moderate number of users, it is possible to detect the signals using conventional matched filter techniques with an acceptable loss in performance. On the other hand, if there are many users in the system and the power control is not perfect, the performance loss can be substantial. This is the main reason for considering the multiuser detection.

2. CDMA System Models

CDMA is an attractive technology for wireless communications. In a CDMA system, each user is assigned a distinct signature sequence (or waveform), which is used to modulate and spread the narrow band information signal over the whole bandwidth. The signature sequences are also known at the receiver in order to demodulate the information signal transmitted by multiple users of the channel, who transmit simultaneously and asynchronously. In section 2.1, a basic CDMA model for synchronous and asynchronous modes is presented. The output of matched filter and the error probability for synchronous and asynchronous models are given in section 2.2 and 2.3, respectively. Section 2.4 introduce other performance measures which is useful for analyzing and evaluating multiuser detectors.

2.1 A Basic CDMA Model

We will consider BPSK transmission through a common Additive White Gaussian Noise (AWGN) channel shared by K simultaneous users employing a DS-CDMA system. As mentioed earlier, each user is assigned a unique signature waveform $s_k(t)$ of duration T, where T is the symbol duration. A signature waveform can be expressed as

$$s_k(t) = \sum_{n=0}^{N-1} a_k(n) h(t - nT_c), \qquad 0 \le t \le T \tag{2.1}$$

where [$a_k(n) \in \{+1, -1\}$, $0 \le n \le N-1$] is a pseudo-noise (PN) code sequence of the k th user consisting of N chips, $h(t)$ is the spreading chip whose duration is $T_c = T/N$. The signature waveforms are assumed to be zero outside the interval [$0,T$], and, therefore, there is no intersymbol interference.

In general, one can assume without loss of generality that all K signature waveforms are normalized so as to have unit energy, i.e.,

$$\left\| s_k \right\|^2 = \int_0^T s_k^2(t)dt = 1 \qquad (2.2)$$

Since BPSK is used in DS-CDMA systems, the information sequences of the k th users denoted by $\{b_k(m)\}$ will have the value of either +1 or –1. Suppose the k th users sends the data block of length M as an input vector

$$\mathbf{b}_k = [b_k(1) \quad b_k(2) \quad \cdots \quad b_k(M)]^T \qquad (2.3)$$

The corresponding base-band transmitted signal of each user can be expressed as

$$g_k(t) = A_k \sum_{i=1}^{M} b_k(i) s_k(t - iT) \qquad (2.4)$$

where A_k is the received amplitude of the k th user's signal such that A_k^2 is referred to as the energy of the k th user. Figure 2.1 illustrates the received waveform, $y(t)$, comprising the sum of K transmitted waveforms in AWGN which can be expressed as

$$y(t) = \sum_{k=1}^{K} A_k \sum_{i=1}^{M} b_k(i) s_k(t - iT - \tau_k) + \sigma n(t) \qquad (2.5)$$

where τ_k is the transmission delay and $n(t)$ is white Gaussain noise with unit power spectral density. Note that the noise power in a frequency band with bandwidth B is $2\sigma^2 B$ (the noise one-sided spectral density level $2\sigma^2$ is often denoted by N_0).

Without loss of generality, we can assume that the delays τ_k are smaller than the bit period time T ($0 \leq \tau_k \leq T$ for $1 \leq k \leq K$) and $0 \leq \tau_1 \leq \tau_2 \leq \ldots \leq \tau_k < T$. Additionally, we shall assume that the data rate $1/T$ is identical for all users. As a result, the Eq.(2.5) represents the model of the multiuser transmitted signal in an asynchronous mode.

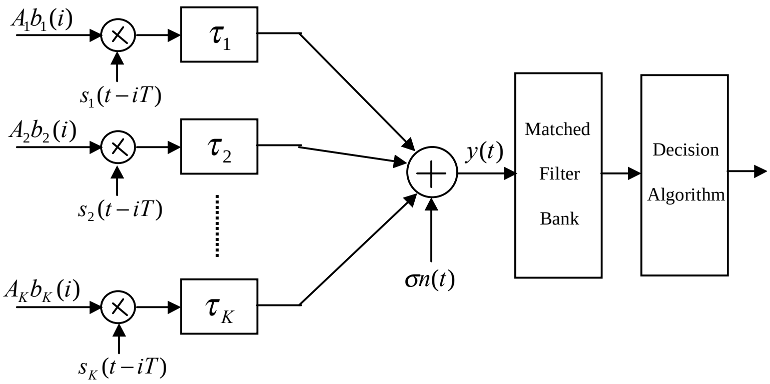

Figure 2.1 Model of the multiuser transmitted signal in an asynchronous mode.

The performance of an asynchronous CDMA system depends on the cross-correlation between every pair of the signature waveforms. In asynchronous DS-CDMA systems, every symbol of a given user overlaps with the two consecutive symbols of each of the interferers as shown in Figure 2.2. Hence, in an asynchronous case, we define two cross-correlations between every pair of signature waveforms. If $k < l$, then we denote

$$\begin{aligned}\rho_{kl}(\tau) &= \int_{\tau}^{T} s_k(t)s_l(t-\tau)dt \\ \rho_{lk}(\tau) &= \int_{0}^{\tau} s_k(t)s_l(t+T-\tau)dt\end{aligned} \qquad (2.6)$$

where $\tau \in [0,T]$ and T is the bit duration.

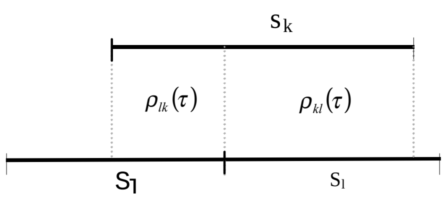

Figure 2.2: Asynchronous cross-correlation.

In a synchronous DS-CDMA system, individual user signals are symbol synchronized. It is then sufficient to consider the one-shot version of Eq.(2.5). Consequently, the received waveform signal in a synchronous DS-CDMA system can be obtained as a special case of Eq. (2.5), where the transmission delay $\tau_k = 0$ for all k, i.e.,

$$y(t) = \sum_{k=1}^{K} A_k b_k s_k(t) + \sigma n(t) \qquad (2.7)$$

and the "synchronous" cross-correlation is given by

$$\rho_{kl} = \int_0^T s_k(t) s_l(t) dt \qquad (2.8)$$

2.2 Matched Filter Outputs for Synchronous and Asynchronous Models

Multiuser detectors basically have a front-end whose objective is to convert the received continuous-time waveform, *y(t)*, into a discrete-time process. This is done by passing the received waveform through a bank of matched filters as depicted in Figure 2.3, each matched to the signature waveform of a different user. The matched filter output is then given by

$$\begin{aligned} y_1 &= \int_0^T y(t) s_1(t) dt \\ &\vdots \\ y_K &= \int_0^T y(t) s_K(t) dt \end{aligned} \qquad (2.9)$$

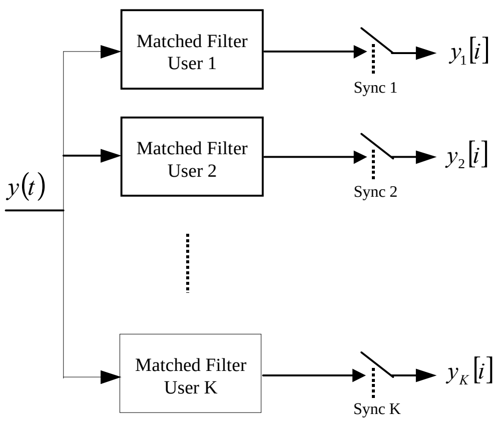

Figure 2.3: Discrete-time K-dimensional vector of matched filter outputs.

In the synchronous case, it is sufficient to restrict our attention to a one-shot model. Therefore, one can express the output of the *k* th matched filter as

$$
\begin{aligned}
y_k &= \langle y, s_k \rangle \\
&= \int_0^T y(t) s_k^*(t)dt \\
&= A_k b_k + \sum_{j \neq k} A_j b_j \rho_{kj} + n_k \qquad (2.10) \\
&= \text{Desired information + MAI + Noise}
\end{aligned}
$$

where ρ_{kj} is defined in Eq.(2.8) and n_k is given by

$$n_k = \sigma \int_0^T n(t) s_k(t)dt \qquad (2.11)$$

is a Gaussian random variable with zero mean and variance equal to σ^2. It is convenient to express Eq.(2.10) in matrix form as follows:

$$\mathbf{y} = \mathbf{RAb} + \mathbf{n} \qquad (2.12)$$

where **R** is the normalized cross-correlation matrix given by

$$
\mathbf{R} = \begin{bmatrix} 1 & \rho_{12} & \cdots & \rho_{1K} \\ \rho_{21} & 1 & \cdots & \rho_{2K} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{K1} & \rho_{K2} & \cdots & 1 \end{bmatrix}
$$

$$\mathbf{y} = [y_1 \cdots \quad y_K]^T$$

$$\mathbf{b} = [b_1 \cdots \quad b_K]^T$$

$$\mathbf{A} = diag\{A_1 \quad \cdots \quad A_K\}$$

and **n** is a zero-mean Gaussian random vector with covariance matrix equal to

$$E[\mathbf{nn}^T] = \sigma^2 \mathbf{R} \qquad (2.13)$$

For an asynchronous channel, the matched filter output for the *k* th user is given by [2]

$$y_k[i] = A_k b_k[i] + \sum_{j<k} A_j b_j[i+1]\rho_{kj} + \sum_{j<k} A_j b_j[i]\rho_{jk} + \sum_{j>k} A_j b_j[i]\rho_{kj} + \sum_{j>k} A_j b_j[i-1]\rho_{jk} + n_k[i] \qquad (2.14)$$

where

$$n_k[i] = \sigma\int_{\tau_{k+iT}}^{\tau_k+iT+T} n(t)s_k(t-iT-\tau_k)dt \qquad (2.15)$$

We can write the above equation in matrix form as

$$\mathbf{y}[i] = \mathbf{R}^T[1]\mathbf{A}\mathbf{b}[i+1] + \mathbf{R}[0]\mathbf{A}\mathbf{b}[i] + \mathbf{R}[1]\mathbf{A}\mathbf{b}[i-1] + \mathbf{n}[i] \qquad (2.16)$$

where the zero-mean Gaussian process **n**[i] has the autocorrelation matrix

$$\mathrm{E}\left[\mathbf{n}[i]\mathbf{n}^T[i]\right] = \begin{cases} \sigma^2\mathbf{R}^T[1], & \text{if } j = i+1; \\ \sigma^2\mathbf{R}[0], & \text{if } j = i; \\ \sigma^2\mathbf{R}[1], & \text{if } j = i-1; \\ 0, & \text{otherwise.} \end{cases} \qquad (2.17)$$

and the matrices **R**[0] and **R**[1] are defined as

$$R_{jk}[0] = \begin{cases} 1 & \text{if } j = k; \\ \rho_{jk} & \text{if } j < k; \\ \rho_{kj} & \text{if } j > k. \end{cases} \qquad (2.18)$$

$$R_{jk}[1] = \begin{cases} 0 & \text{if } j \ge k; \\ \rho_{\mathrm{kj}} & \text{if } j < k. \end{cases} \qquad (2.19)$$

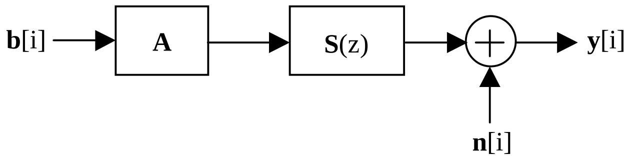

Figure 2.4: Z-domain representation of asynchronous DS-CDMA system.

The matched filter output in Eq.(2.16) can be represented in the z-transform domain as shown in Figure 2.4, where

$$\mathbf{S}(z) = \mathbf{R}^T[1]z + \mathbf{R}[0] + \mathbf{R}[1]z^{-1} \qquad (2.20)$$

2.3 Probability of Error for Synchronous and Asynchronous Users

As derived before for the synchronous case, the k-th user matched filter output is

$$y_k = A_k b_k + \sum_{j \neq k}^{K} A_j b_j \int_0^T s_j(t) s_k(t) dt + n_k \qquad (2.21)$$

where

$$n_{k=} \int_0^T s_k(t) n(t) dt \qquad (2.22)$$

is a Gaussian random variable with zero mean and variance equal to σ^2. If the signature waveform of the *k* th user is orthogonal to all the others, resulting in $\rho_{jk} = 0$ for j ≠ k, then the MF output for user k reduces to that obtained in the single user problem:

$$y_k = A_k b_k + n \qquad (2.23)$$

The probability of error of a threshold comparison of y_k is $P = Q(A/\sigma)$, which can be derived as follows. Given only one user, the channel becomes

$$y(t) = Abs(t) + \sigma n(t), \qquad t \in [0,T] \tag{2.24}$$

where the deterministic signal s has unit energy, *n(t)* is Gaussian white noise, and $b \in \{-1,+1\}$. Before we proceed further, let us consider the restricted class of linear detectors. Consider a demodulator that outputs the sign of the correlation of an observed waveform and a deterministic signal h of duration T:

$$\hat{b} = \operatorname{sgn}(\langle y, h \rangle) = \operatorname{sgn}\left(\int_0^T y(t)h(t)dt \right) \tag{2.25}$$

This detector summarizes the information contained in the observed *y(t)* by the scalar decision statistic, <y, h>. To optimize the choice of h, we notice that linearity of the decision statistic makes it easy to discern the respective contributions of signal and noise:

$$Y = <y, h> = Ab<s, h> + \sigma<n, h> \tag{2.26}$$

The distribution of the second term on the right side is given by the following results, which concern the properties of white noise. If h and g are finite-energy deterministic signals and n(t) is white noise with unit spectral density, then

1. $E[<n,h>] = 0$
2. $E[<n,h>^2] = |h|^2$
3. If n(t) is a Gaussian process, then <n, h> is a Gaussian random variable.
4. $E[<n,h><n,g>] = <g,h>$
5. $E[<n,h>n] = h$

A sensible way to maximize signal-to-noise ratio is to choose h so as to make <s, h> large and $\sigma|h|$ small, i.e.,

$$\max_h \frac{A^2(<s,h>)^2}{\sigma^2|h|^2} \tag{2.27}$$

The solution to this problem can be obtained from the Cauchy-Schwarz inequality (<s,h>)^2 $\leq$ $|h|^2|s|^2$, with equality sign when h is a multiple of s. Therefore, all positive multiples of s are equivalent and result in the same decision:

$$\hat{b} = \text{sgn}\left(\langle y, \alpha s\rangle\right) = \text{sgn}\left(\int_0^T y(t)h(t)dt\right) \tag{2.28}$$

where α = any positive integer. This linear detector is known as the *matched filter detector* for signal s. It can be implemented either as a correlator (multiplication of received waveform with s followed by integration) or as a linear filter with impulse response s(T-t) sampled at time T. We have seen that the match filter is optimal in the sense that it maximizes the signal to noise ratio of the statistic Y. notice that the earlier derivation did not assume the white noise to be Gaussian. If we do assume so, we can actually prove that among all linear detectors, the match filter minimizes the probability of error.

From Eq.(2.26), the statistic Y conditioned on b = -1 is a Gaussian distributed random variable with mean -A<,s,h> and variance $\sigma^2|h|^2$, abbreviated as N(-A<s, h>, $\sigma^2|h|^2$). By the same token, the distribution of Y conditioned on b = 1 is N(A<s, h>, $\sigma^2|h|^2$). Figure 2.5 shows both conditional distributions $f_{Y|1}$ and $f_{Y|-1}$.

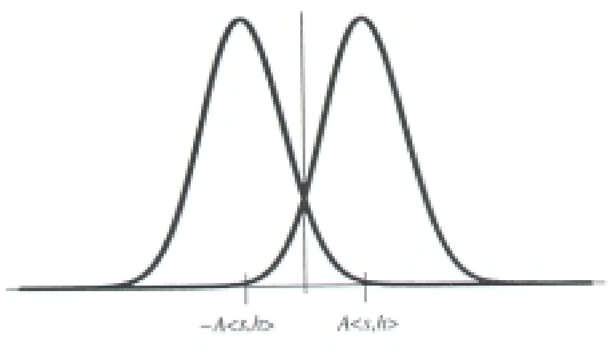

Figure 2.5: Conditional distributions of Y, given b = -1 and b = +1.

The probability of errors can be expressed as follows, with change of integration variable and symmetry:

$$P = \frac{1}{2}\int_{0}^{\infty} f_{Y/-1}(v)dv + \frac{1}{2}\int_{-\infty}^{0} f_{Y/1}(v)dv$$

$$= Q\left(\frac{A < s,h >}{\sigma|h|}\right)$$

$$= Q\left(\frac{A}{\sigma}\right) \tag{2.29}$$

Let us now consider a two-user *non-orthogonal* CDMA case as shown in Figure 2.6 and 2.7. The probability of error for user 1 is

$$P_1^c(\sigma) = P[b_1 \neq \hat{b}_1]$$

$$= P[b_1 = +1]P[y_1 < 0|b_1 = +1] + P[b_1 = -1]P[y_1 > 0|b_1 = -1]$$

$$P[y_1 > 0|b_1 = -1] = P[y_1 > 0|b_1 = -1, b_2 = +1]P[b_2 = +1] + P[y_1 > 0|b_1 = -1, b_2 = -1]P[b_2 = -1]$$

$$P[y_1 > 0|b_1 = -1] = P[n_1 > A_1 - A_2\rho]P[b_2 = +1] + P[n_1 > A_1 + A_2\rho]P[b_2 = -1]$$

$$P[y_1 > 0|b_1 = -1] = \frac{1}{2}Q\left(\frac{A_1 - A_2\rho}{\sigma}\right) + \frac{1}{2}Q\left(\frac{A_1 + A_2\rho}{\sigma}\right) \tag{2.30}$$

By symmetry, P[$y_1 < 0$ | $b_1 = +1$] = P[$y_1 > 0$ | $b_1 = -1$]. It then follows that the bit-error-rate (BER) of the conventional receiver for user 1 in the presence of another interfering user is

$$P_1^c(\sigma) = \frac{1}{2}Q\left(\frac{A_1 - A_2\rho}{\sigma}\right) + \frac{1}{2}Q\left(\frac{A_1 + A_2\rho}{\sigma}\right) \tag{2.31}$$

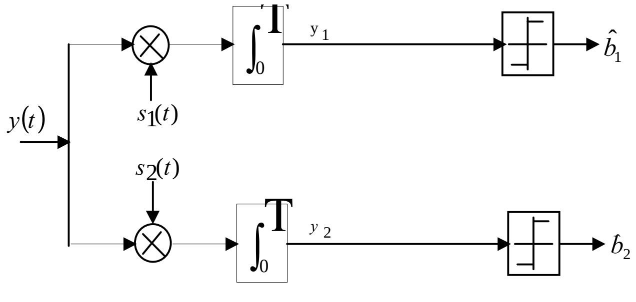

Figure 2.6: Bank of single-user matched filter (K=2).

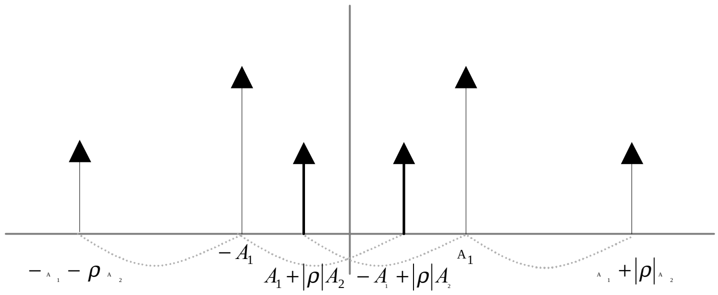

Figure 2.7: Output of matched filter for user 1 with one interference.

Interchanging the roles of users 1 and 2, one obtain

$$P_2^c(\sigma) = \frac{1}{2}Q\left(\frac{A_2 - A_1\rho}{\sigma}\right) + \frac{1}{2}Q\left(\frac{A_2 + A_1\rho}{\sigma}\right) \tag{2.32}$$

Since the Q-function is monotonically increasing, we can determine the upper bound

$$P_1^c(\sigma) \le \frac{1}{2}Q\left(\frac{A_1 - A_2|\rho|}{\sigma}\right) \tag{2.33}$$

The generalization of BER from the 2-user case to an arbitrary number of users is straightforward. Following the same reasoning as before, we can write BER of the k th user as:

$$\begin{aligned} P_k^c(\sigma) &= P[b_k = +1]P[y_k < 0 | b_1 = +1] + P[b_k = -1]P[y_k > 0 | b_k = -1] \\ &= \frac{1}{2}P[n_k > A_k - \sum_{j\neq k} A_j b_j \rho_{jk}] + \frac{1}{2}P[n_k < -A_k - \sum_{j\neq k} A_j b_j \rho_{jk}] \\ &= P[n_k > A_k - \sum_{j\neq k} A_j b_j \rho_{jk}] \end{aligned} \tag{2.34}$$

Similar to the 2-user case, BER for the *k* th user case is upper bounded by

$$P_k^c \le Q\left(\frac{A_k}{\sigma} - \sum_{j \ne k} \frac{A_j}{\sigma} \left|\rho_{jk}\right|\right) \tag{2.35}$$

Since the exact expression for BER grows exponentially with the number of users, it is frequently approximated by

$$\tilde{P}_k^c(\sigma) = Q\left(\frac{A_k}{\sqrt{\sigma^2 + \sum_{j \ne k} A_j^2 \rho_{jk}^2}}\right) \tag{2.36}$$

This approximation works well at low SNR, but may be unreliable at high SNR.

The analysis in the asynchronous case can be done in a similar fashion. The major difference is that each bit is affected by 2K-2 interfering bits instead of K-1 bits as in the synchronous case. Probability of error for the k-th user is

$$P_k^c(\sigma) = \frac{1}{2^{2K-2}} \sum_{1} \cdots \sum_{j \ne k} \cdots \sum_{K} Q\left(\frac{A_k}{\sigma} + \sum_{j \ne k} \frac{A_j}{\sigma} (e_j \rho_{jk} + d_j \rho_{kj})\right) \tag{2.37}$$

Where e_j and $d_j \in \{-1, +1\}$. The asynchronous crosscorrelations in the above equation depend on the offsets between the users' symbol periods. As a result, those parameters are random variables that may actually be time varying. Given a set of signature waveforms, it is possible to compute the distribution or at least the expectation of $P_k^c(\sigma)$. However, this is a computationally intensive task [3].

2.4 Asymptotic Multiuser Efficiency and Related Measures

The bit error rate is widely used to measure the performance of the communication systems of interest. In the concept of multiuser detection, there are, however, other performance measures derived from the bit error rate that are useful in the analysis and design. We shall introduce some performance measures necessary for analyzing multiuser detectors.

2.4.1 Signal-to-Interference Ratio (SIR)

The signal-to-interference ratio is useful in an assessment of the quality of multiuser detectors. SIR is the ratio between the powers of the desired user and those of all other components at the output of the single-user matched filter. In the absence of interfering users, the output SIR of the k th user is given by A_k^2/σ^2. For synchronous DS-CDMA systems, the output SIR of the k th user is

$$SIR = \frac{A_k^2}{\sigma^2 + \sum_{j \neq k} A_j^2 \rho_{jk}^2} \tag{2.38}$$

2.4.2 The effective energy

It is useful to quantify the multiuser error probability with respect to the optimum single user error probability. The effective energy of the k th user, denoted by $e_k(\sigma)$, is defined as the energy that user k requires to achieve bit error rate equal to $P_k(\sigma)$ in a single user Gaussian channel with the same background noise level, i.e.,

$$P_k(\sigma) = Q\left(\frac{\sqrt{e_k(\sigma)}}{\sigma}\right) \tag{2.39}$$

Since the error probability of any multiuser detector is lower bounded by the single user error probability

$$P_k(\sigma) \geq Q\left(\frac{A_k}{\sigma}\right), \tag{2.40}$$

the effective energy is always upper bounded by the actual energy, i.e.,

$$e_k(\sigma) \leq A_k^2 \tag{2.41}$$

The one-to-one correspondence between $e_k(\sigma)$ and $P_k(\sigma)$ is given by

$$e_k(\sigma) = \sigma^2 \left(Q^{-1}(P_k(\sigma))\right)^2 \tag{2.42}$$

2.4.3 The multiuser efficiency

The multiuser efficiency is the ratio between the effective and actual energy, $e_k(\sigma)/A_k^2$. It can also be used to characterize the multiuser bit error rate and quantify the performance loss due to the existence of other users in the channel. From Eq.(2.24), it is clear that the value of multiuser efficiency is in the interval [0, 1]. Multiuser efficiency depends upon signature waveforms, received signal-to-noise ratios, and types of detectors.

2.4.4 The asymptotic multiuser efficiency

The asymptotic multiuser efficiency is defined as

$$\eta_k = \lim_{\sigma \to 0} \frac{e_k(\sigma)}{A_k^2} \tag{2.43}$$

and measures the slope with which $P_k(\sigma)$ goes to 0 (in logarithmic scale) in the high signal-to-noise region; that is,

$$\eta_k = \sup\left\{0 \le r \le 1: \quad \lim P_k(\sigma)/Q\left(\frac{\sqrt{r}A_k}{\sigma}\right) = 0\right\} \tag{2.44}$$

$$= \frac{2}{A_k^2} \lim_{\sigma \to 0} \sigma^2 \log 1/P_k(\sigma) \tag{2.45}$$

In general, the multiuser efficiency is easier to compute than the bit error rate, and it gives easy-to-grasp illustrations of the effects of unequal received energies.

2.4.5 The near-far resistance

The near-far resistance is used to quantify the degree of robustness of multiuser detectors against the near-far problem. It is defined as the multiuser asymptotic efficiency minimized over received energies of all the other users [4]:

$$\overline{\eta} = \inf_{\substack{A_j > 0 \\ j \neq k}} \eta_k \qquad (2.46)$$

The near-far resistance depends on the signature waveforms and on the demodulators.

3. Multiuser Detection

This chapter contains a brief explanation of multiuser detection for a DS-CDMA system. Treated are both the asynchronous and synchronous cases. The detector presented in the report is mainly based on the linear Minimum Mean Square Error (MMSE) detector.

3.1 Background

Multiuser detection concerns the detection of information sent simultaneously by several transmitters sharing a multiple-access channel. A more challenging channel sharing strategy is the CDMA approach, where all users share the same time and frequency band. To distinguish between the different users, a unique waveform (code) is assigned to each user. Therefore, the received signal from all users is a superposition of the individually transmitted signals. As a result, the task of the multiuser detector is to reliably demodulate the information from a specific user.

The conventional detector used in single-user systems is the matched filter receiver. It is well known that this receiver is optimal in the minimum of probability of error sense in demodulation of a single existing user in AWGN environment. In demodulation of a user in CDMA system, the noise components from the different matched filters are not uncorrelated due to the cross-correlation between users. Hence, the colored noise should be taken into consideration in demodulation.

The performance of the conventional detector can be acceptable, when the received signals have the similar energies and the waveforms have low cross-correlation. Basically, the user with high power makes detection of users with low power impossible. This is known as the near-far effect problem. To combat this problem it is required to use accurate power control and design waveforms with as low cross-correlation as possible. However, it is typically not possible to design codes that are orthogonal in the receiver, either because the transmissions from different users are uncoordinated (asynchronous) or that the channel is a multipath channel. As a consequence, in asynchronous and multipath CDMA systems, the conventional receiver will always suffer from the near-far effect, and the performance will be limited by the interference from other users. Multiuser detection is then a good method to cope with the problem of near-far effects and the interference limited performance.

3.2 Linear Detector

In general, there exist many optimum detectors for a DS-CDMA system, but they usually require a high computational complexity. For example, the complexity of the maximum likelihood (ML) detector grows exponentially with the number of users. One class of detectors with substantially lower complexity is the linear filter detectors. For these detectors, the complexity increases linearly with the number of users. There are two well known linear detectors, i.e., the decorrelator detector and the MMSE detector. The former will decorrelate the channel making the information of interest orthogonal to the interference. Nevertheless, its major drawback is the noise enhancement due to the inverse filtering. The latter does not completely remove the interference, which for low signal-to-noise ratios results in better performance since the noise is not enhanced in the same way as the decorrelator does. It has been shown that, for very low signal-to-noise ratios the MMSE detector converges to the conventional receiver, and for the high signal-to-noise ratios it converges to the decorrelator. We shall then focus on the MMSE linear multiuser detector

3.3 MMSE Linear Multiuser Detector

In the estimation theory, it is a problem of estimating a random variable W based on a given set of observation Z. This can be done by choosing the function $\hat{W}(Z)$ that minimizes the mean-square error (MSE), i.e.,

$$E\left[\left(W-\hat{W}(Z)\right)^2\right] \qquad (3.1)$$

The solution to this problem is the conditional-mean estimator:

$$\hat{W}(Z) = E[W \mid Z] \tag{3.2}$$

Basically, it requires the joint distribution of W and Z to obtain $\hat{W}(Z)$. To avoid this difficulty, one can minimize MSE within the restricted set of linear transformations of Z. Then, the linear minimum MSE (MMSE) estimator is easy to compute and it depends on the joint distribution of W and Z through only their variance and covariance.

Recall from Eq.(2.13), where $\mathbf{y} = \mathbf{RAb} + \mathbf{n}$, one can notice that the matched filter output of the k th user, y_k, depends only on the transmitted bits, the signature cross-correlation and the noise. Therefore **y** is a sufficient statistic for estimating the transmitted bits, **b**. As a result, the decision algorithm for the MMSE detector can be implemented into two steps [5]. First a linear estimate $\hat{\mathbf{b}} = \mathbf{My}$ of the transmitted bits **b** is obtained from the matched filter output **y**. Second the estimate $\hat{\mathbf{b}}$ is compared to a threshold of zero, and the final bit estimates are obtained to be the value of sgn($\hat{\mathbf{b}}$). The best estimate **M** linear transformation can be obtained by minimizing

$$\min_{\hat{\mathbf{b}}} E\left[\left(\mathbf{b} - \hat{\mathbf{b}}\right)^T \left(\mathbf{b} - \hat{\mathbf{b}}\right)\right] \tag{3.3}$$

or, equivalently,

$$\min_{\mathbf{M}} E\left[\left\|\mathbf{b} - \mathbf{My}\right\|^2\right] \tag{3.4}$$

Since

$$\left\|\mathbf{x}\right\|^2 = trace\left\{\mathbf{x}\mathbf{x}^T\right\} \tag{3.5}$$

It can be shown from [2] that

$$\min_{\mathbf{M}} E\left[\left\|\mathbf{b} - \mathbf{My}\right\|^2\right] = \min_{\mathbf{M}} trace\left\{\left[\mathbf{I} + \sigma^{-2}\mathbf{ARA}\right]^{-1}\right\} \tag{3.6}$$

As a result, the decision output of the MMSE linear detector can be obtained by

$$\hat{b}_k = \text{sgn}\left(\frac{1}{A_k} \left(\left[\mathbf{R} + \sigma^2 \mathbf{A}^{-2} \right]^{-1} \mathbf{y} \right)_k \right)$$
$$= \text{sgn}\left(\left(\left[\mathbf{R} + \sigma^2 \mathbf{A}^{-2} \right]^{-1} \mathbf{y} \right)_k \right) \quad (3.7)$$

where

$$\sigma^2 \mathbf{A}^{-2} = diag\left\{ \frac{\sigma^2}{A_1^2}, \ldots, \frac{\sigma^2}{A_k^2} \right\} \quad (3.8)$$

Observe that the dependence of the MMSE detector on the received amplitudes is only through the signal to noise ratios (SNR) A_k/σ. Figure 3.1 depicts the MMSE linear detector for the synchronous DS-CDMA system.

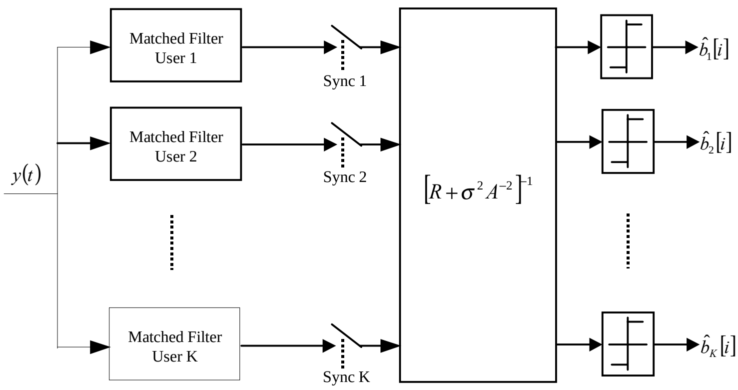

Figure 3.1: MMSE linear detector for a synchronous DS-CDMA system.

Figure 3.2 illustrates MMSE linear detection for two synchronous users. In this case, we have

$$\left[\mathbf{R} + \sigma^2 \mathbf{A}^{-2} \right] = \left[\left(1 + \frac{\sigma^2}{A_1^2} \right) \left(1 + \frac{\sigma^2}{A_2^2} \right) - \rho^2 \right]^{-1} \begin{bmatrix} 1 + \frac{\sigma^2}{A_2^2} & -\rho \\ -\rho & 1 + \frac{\sigma^2}{A_1^2} \end{bmatrix} \quad (3.9)$$

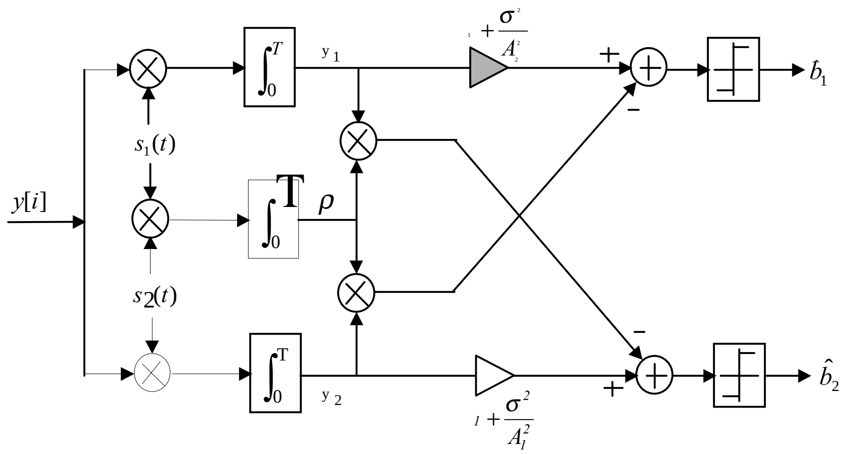

Figure 3.2: The MMSE linear detector for two synchronous users.

As known in the literature, the single-user matched filter detector is optimized to combat the background white noise, while the decorrelating detector eliminates the multiuser interference regardless of the background noise. Therefore, MMSE linear detector can be seen as a compromise solution that takes into account the background noise and the interference of each user. One can consider the conventional detector and the decorrelating detector as a special case of the MMSE linear detector.

If we fix A_1 and let $A_2, \ldots, A_K \rightarrow 0$, then the first row of $\left[\mathbf{R} + \sigma^2 \mathbf{A}^{-2}\right]^{-1}$ becomes

$$\left[\frac{A_1^2}{A_1^2 + \sigma^2}, 0, \cdots, 0\right] \tag{3.10}$$

which corresponds to the matched filter for user 1. As σ increases, $\left[\mathbf{R} + \sigma^2 \mathbf{A}^{-2}\right]^{-1}$ becomes a strongly diagonal matrix. Accordingly, the MMSE detector approaches the conventional detector as $\sigma \rightarrow \infty$. On the other hand, if we hold all the amplitudes fixed and let $\sigma \rightarrow 0$, then

$$\left[\mathbf{R} + \sigma^2 \mathbf{A}^{-2}\right]^{-1} \rightarrow \mathbf{R}^{-1} \tag{3.11}$$

As a result, the MMSE linear detector converges to the decorrelating detector as the signal-to-noise ratios go to infinity. This fact implies that the MMSE linear detector has the same asymptotic efficiency and near-far resistance as the decorrelating detector. Moreover, the MMSE linear detector also achieve optimum near-far resistance.

In asynchronous DS-CDMA systems, the MMSE linear detector is K-input, K-output, linear, time-invariant filter with transfer function

$$[\ \mathbf{R}^T[1]z + \mathbf{R}[0] + \sigma^2\mathbf{A}^{-2} + \mathbf{R}[1]z^{-1}\ ]^{-1} \tag{3.12}$$

Note that the inverse matrix in Eq.(3.12) always exists because it is the sum of a nonnegative definite matrix and the diagonal positive definite matrix $\sigma^2\mathbf{A}^{-2}$.

4. Implementations of the MMSE Linear Detector

This chapter discusses practical ways to implement the linear MMSE detection. They include adaptive MMSE and blind MMSE multiuser detection algorithms.

4.1 Adaptive MMSE linear Multiuser Detection

The adaptive MMSE linear multiuser detection scheme is attractive mainly because of its ease of implementation. This adaptive MMSE detection method does not require on-line computation of impulse response, knowledge of crosscorrelations, and in general, the signature waveforms of interfering users. The adaptive implementation of MMSE can learn the desired filter impulse response from the received waveform, provided that the data of the desired user is known to the receiver. In practice, this scheme requires transmission of a training sequence, a string of data known to the receiver, prior to the transmission of actual data. The receiver uses an adaptive law to adjust its linear transformation while the training sequence is in transmission. If correlations and amplitudes vary over time, training sequences can be sent periodically to readjust the receiver. It is also common to perform fine adjustment of the linear transformation (once the adaptive algorithm has converged and the transmission of training sequence is completed) by letting the adaptive algorithm run with decisions made by the detector in stead of the true transmission data.

We will use the steepest decent method to derive the adaptive MMSE detection. Before applying the method, a review of the elements of gradient descent stochastic optimization should be in order at this point. The function (convex) to be minimized, with respect to θ, is

$$\Psi(\theta) = E[g(X,\theta)] \tag{4.1}$$

where X is a random variable. Start off with θ_0, and use the recursive equation

$$\theta_{j+1} = \theta_j - \mu\nabla\psi(\theta_j) \tag{4.2}$$

Replace the unknown $\nabla\psi(\theta_j) = E\nabla g(X,\theta_j)$ with its "noisy" version $\nabla g(X_{j+1},\theta_j)$. The new recursive equation is

$$\theta_{j+1} = \theta_j - \mu\nabla g(X_{j+1},\theta_j) \tag{4.3}$$

4.1.1 Adaptive MMSE in the synchronous case

Let us now apply the forgoing general framework to the derivation of an adaptive MMSE linear detector in the synchronous case. The MMSE linear detector for user 1 correlates the received waveform with signal c_1 that minimizes

$$E[\,(b_1 - \langle c_1,y\rangle)^2\,] \tag{4.4}$$

The minimization fits the stochastic framework with

$$g(X,c_1) = (b_1 - \langle c_1,y\rangle)^2 \tag{4.5}$$

The independent and identically distributed observations used in the stochastic approximation are $X_j = (b_1[j], y[j])$, where

$$y[j](t) = \sum_{i=1}^{K} A_i b_i[j] s_i(t - jT) + n(t), t \in [jT, (j+1)T] \tag{4.6}$$

Therefore, all we need to do to specify the gradient descent algorithm is to obtain the gradient of $(b_1 - \langle c_1,y\rangle)^2$ with respect to c_1, which equals $2(\langle c_1,y\rangle - b_1)y$. It follows that the adaptive algorithm is

$$c_1[j] = c_1[j-1] - \mu(\langle c_1[j-1], y[j]\rangle, y[j] - b_1[j])\, y[j] \tag{4.7}$$

In words, the update of impulse response is equal to the received waveform scaled by the error between the known data and filter output. Using matrix notations, with the sequence of received vectors being

$$\mathbf{r}[n] = \mathbf{SAb}[n]+\sigma\mathbf{m}[n] \quad (4.8)$$

where **m** is an L-dimensional Gaussian vector with independence unit variance components, and **S** LxK matrix of signature vectors **S** = [$\mathbf{s}_1$ $\mathbf{s}_2$ …. $\mathbf{s}_k$]. The finite-dimensional version of the adaptive algorithm becomes

$$\mathbf{v_1}[n] = \mathbf{v_1}[n-1] - \mu(\mathbf{v_1}^T[n-1]\mathbf{r}[n] - b_1[n])\,\mathbf{r}[n] \quad (4.9)$$

It can be proved that this algorithm converges globally when μ decreases suitably. To ensure stability, step size must be smaller than $\mu_{max} = 2/(\sigma^2+\lambda_{max})$, where λ_{max} is the maximum eigenvalue of

$$\sum_{k=1}^{K} A_k^2 \, s_k s_k^T \quad (4.10)$$

If the MMSE filter corresponds to

$$\mathbf{v_1}^* = E[\mathbf{rr}^T]^{-1}\, E[b_1\mathbf{r}] \quad (4.11)$$

then the expected error the adaptive filter and the desired solution is

$$\begin{aligned} E[\mathbf{v}_1[n]-\mathbf{v}_1^*] &= E[\mathbf{v}_1[n-1]-\mathbf{v}_1^*] - \mu E[\mathbf{r}[n]\mathbf{r}^T[n]\mathbf{v}_1[n-1]] + \mu E[\mathbf{rr}^T]\mathbf{v}_1^* \\ &= [\mathbf{I}-\mu E[\mathbf{rr}^T]]E[\mathbf{v}_1[n-1]-\mathbf{v}_1^*] \end{aligned} \quad (4.12)$$

4.1.2 Adaptive MMSE in the Asynchronous case

If we take a one-shot approach, the adaptive MMSE in the asynchronous case will be the same as that in the synchronous case, i.e., the adaptive algorithm holds verbatim in both cases. The expected error, however, is different in the asynchronous case because of the dependence of the observations in the adjacent blocks.

4.2 Blind MMSE Multiuser Detection

Adaptive algorithms that operate without knowledge of channel inputs are called *blind*. The blind MMSE detector does not require sequence training but does need signature waveform and timing of the desired user. Let us define

$$MOE(x_1) = E[(\langle y,c_1\rangle)^2] = E[(\langle y,s_1+x_1\rangle)^2] \tag{4.13}$$

Here we express c_1, using *canonical representation*, as s_1+x_1, where $\langle s_1,x_1\rangle = 0$. The unconstrained gradient of $MOE(x_1)$ is

$$\nabla MOE = 2\langle y,s_1+x_1\rangle y \tag{4.14}$$

The component in this gradient orthogonal to s_1 is a scaled version of the component of y orthogonal to s_1:

$$y - \langle y,s_1\rangle s_1 \tag{4.15}$$

Therefore the projected gradient (orthogonal to s_1) is

$$2\langle y,s_1+x_1\rangle\,[y-\langle y,s_1\rangle s_1] \tag{4.16}$$

If we denote the responses of the matched filters s_1 and $s_1+x_1[i-1]$ by $Z_{MF}[i] = \langle y[i],s_1\rangle$ and $Z[i] = \langle y[i],s_1+x_1[i-1]\rangle$, respectively. Then the stochastic gradient adaptive rule is

$$x_1[i] = x_1[i-1] - \mu\, Z[i](y[i] - Z_{MF}[i]s_1) \tag{4.17}$$

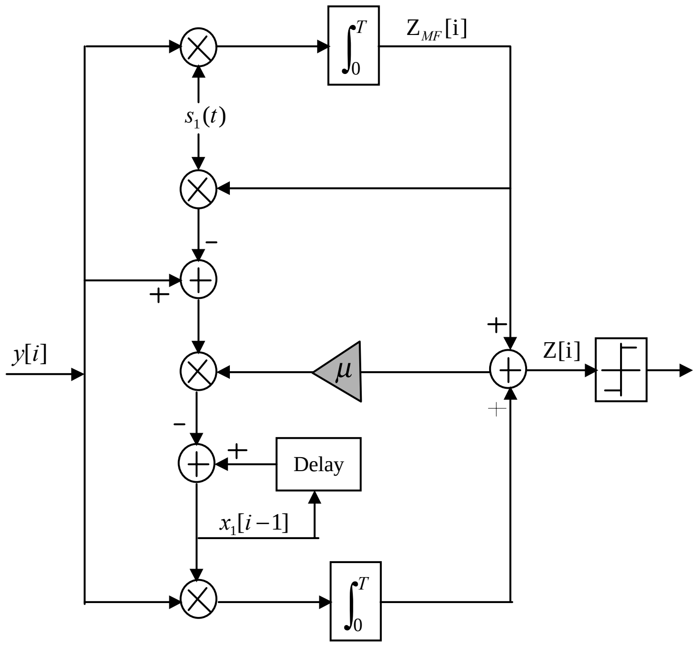

Figure 4.1: Blind adaptive multiuser detection.

This adaptive MMSE algorithm is depicted in Figure 4.1. In the absence of information about interfering waveforms (or for ease of implementation the natural choice of the initial condition would be $x_1[i] = 0$. The adaptive rule converges to linear MMSE detector (which achieves maximum near-far resistance) using no more information than the single-user matched filter (which yields zero near-far resistance), namely, the desired user's signature and its timing.

In the practical implementation of this adaptive algorithm, finite precision round-off error may have a cumulative effect that drives the updates well outside of the required orthogonal subspace. This can be remedied by occasionally replacing the update $x_1[i]$ by its orthogonal projection: $x_1[i] - \langle x_1[i], s_1 \rangle s_1$. Regarding the implementation of this adaptive algorithm, we make the following observations [6]:

1. Implementation with finite-dimensional vectors rather than continuous-time signals. For the sake of reducing computational complexity and improved speed of convergence, it is desirable to use vector space with lowest dimension that contains the desired and interfering signals.
2. Improved convergence with more complex recursions.
3. Implementation in asynchronous channels.

5. Performances of the MMSE Linear Detector

Since the MMSE linear detector converges to the decorrelator as $\sigma \to 0$, its near-far resistance and asymptotic multiuser efficiency are identical to those of decorrelator, i.e.,

$$\overline{\eta} = \frac{1}{\mathbf{R}_{kk}^{+}} \tag{5.1}$$

in the synchronous case, and

$$\overline{\eta} = \left(\frac{1}{2\pi} \int_{-\pi}^{\pi} \left[\mathbf{R}^{T}[1] e^{j\omega} + \mathbf{R}[0] + \mathbf{R}[1] e^{-j\omega} \right]_{kk}^{+} d\omega \right)^{-1} \tag{5.2}$$

in the asynchronous case.

In a special case where we have high signal-to-noise ratio channels with linearly independent signature waveforms, the MMSE detector can obtain a very small performance improvement over the decorrelator. However, the key advantage of the MMSE detector is the ease with which it can be implemented adaptively.

In a synchronous case, the first component of the output of the linear MMSE transformation can be expressed as

$$\left(\mathbf{M}^{*} \mathbf{y} \right)_1 = \left(\left[\mathbf{R} + \sigma^2 \mathbf{A}^{-2} \right]^{-1} y \right)_1 = B_1 \left(b_1 + \sum_{k=2}^{K} \beta_k b_k \right) + \sigma \tilde{n}_1 \tag{5.3}$$

where

$$M^{*} = \left[\mathbf{R} + \sigma^2 \mathbf{A}^{-2} \right]^{-1} \tag{5.4}$$

$$\beta_k = \frac{B_k}{B_1} \tag{5.5}$$

$$B_k = A_k \left(\mathbf{M}^* \mathbf{R}\right)_{1k} \tag{5.6}$$

$$\tilde{n}_1 \sim \mathrm{N}\left(0, \left(\mathbf{M}^* \mathbf{R} \mathbf{M}^*\right)_{11}\right) \tag{5.7}$$

The *leakage coefficient* β_k expresses the contribution of the k th interferer to the decision statistic with respect to the that of the desired user. As a result, the error probability is given by

$$P_1^m(\sigma) = 2^{1-K} \sum_{b_2,\ldots,b_K \in \{-1,1\}^{K-1}} Q\left(\frac{A}{\sigma} \frac{\left(M^* R\right)_{11}}{\sqrt{\left(M^* R M^*\right)_{11}}} \left(1 + \sum_{k=2}^{K} \beta_k b_k\right)\right) \tag{5.8}$$

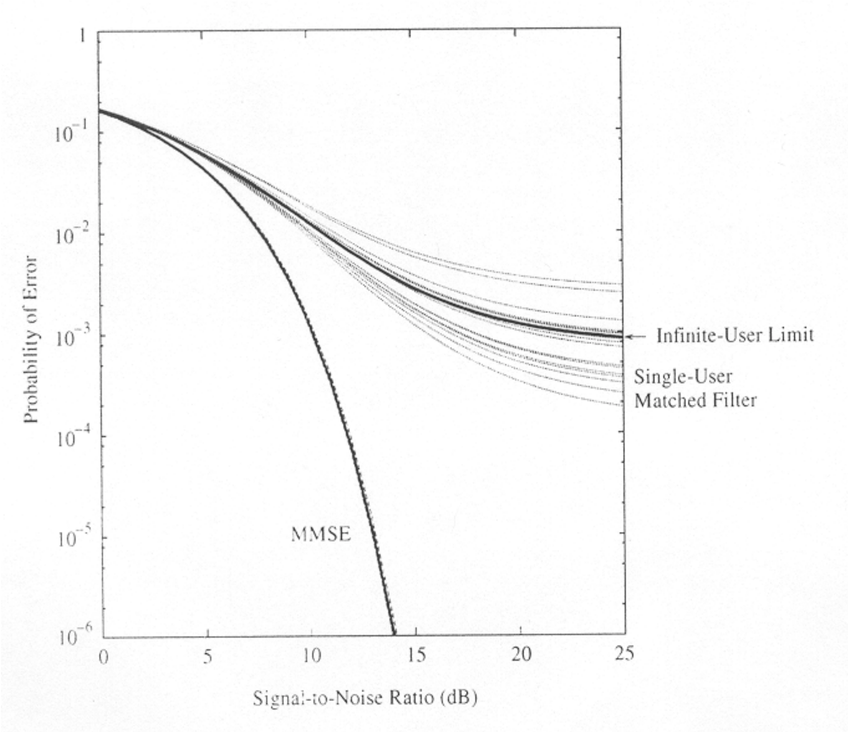

Figure 5.1: Bit-error-rate with two users and cross-correlation ρ = 0.8: (a) Single-user matched filter; (b) Decorrelator; (c) MMSE; (d) Minimum (upper bound); (e) Minimum (lower bound).

Figure 5.1 computes the Eq.(5.8) in the special case of two users with the cross-correlation equal to 0.8. The signal-to-noise ratio of the desired user is equal to 10 dB. The error probability is plotted as a function of the near-far ratio A_2/A_1 and it is also compared to the error probability of the single user matched filter, the decorrelator, and the optimum multiuser detector. Observe that for sufficiently low interferer power, the error probability of MMSE detector is better than that of decorrelator. For slightly high interferer power, the MMSE detector performs similar to the decorrelator and much better than the single user matched filter.

According to the numerical and analytical results from [7], it has shown that the bit-error-rate of the MMSE detector is better than that of the decorrelator for all levels of background Gaussian noise, number of users, and cross-correlation matrices.

Consider the case where random binary-valued signature sequences are used. Recall Eq. (3.6), i.e.,

$$\min_{\mathbf{M}} E\left[\|\mathbf{b}-\mathbf{M}\mathbf{y}\|^2\right] = \min_{\mathbf{M}} trace\left\{\left[\mathbf{I}+\sigma^{-2}\mathbf{A}\mathbf{R}\mathbf{A}\right]^{-1}\right\} \tag{5.9}$$

If all the users have equal power, the MMSE for the k th user is the k th diagonal element of the matrix

$$\left[\mathbf{I}+\frac{A^2}{\sigma^2}\mathbf{R}\right]^{-1} \tag{5.10}$$

By averaging the minimum mean square error with respect to all possible signature waveforms and evaluating it as the number of users goes to infinity $(K \to \infty)$, Verdu [2] has shown that the minimum mean square error with random binary sequences converges to

$$\lim_{K\to\infty} \min_{c_k} E\left[\left(b_k - \langle c_k, y\rangle\right)^2\right] = 1 - \frac{1}{4\beta}\frac{\sigma^2}{A^2}\Psi\left(\frac{A^2}{\sigma^2}, \beta\right) \tag{5.11}$$

where the expectation is with respect to the noise and transmitted data, and

$$\Psi(x,z)=\left(\sqrt{x\left(1+\sqrt{z}\right)^2+1}-\sqrt{x\left(1-\sqrt{z}\right)^2+1}\right)^2 \tag{5.12}$$

Since the signal-to-interference ratio achieved by the MMSE detector is upper bounded by A^2/σ^2, the signal-to-interference ratio converges in mean square sense to

$$\frac{A_{MMSE}^2}{\sigma^2}=\frac{A^2}{\sigma^2+\beta\dfrac{A^2}{1+\dfrac{A_{MMSE}^2}{\sigma^2}}} \tag{5.13}$$

Recall that in the single user matched filter case, the signal-to-interference ratio converges in mean square sense to

$$\lim_{K\to\infty}\frac{A^2}{\sigma^2}=\frac{A^2}{\sigma^2+\beta A^2} \tag{5.14}$$

Consequently, one can see that in the single user matched filter case, every interferer contributes an interfering power equal to its power divided by the spreading gain *N*, while in MMSE processing that interfering power is further reduced by a factor of

$$\frac{1}{1+\dfrac{A_{MMSE}^2}{\sigma^2}} \tag{5.15}$$

6. Conclusions

DS-CDMA is the most commonly proposed CDMA system for the third generation of mobile systems. It has many advantages as explained in section 1.3. However, its performance is mainly degraded by the near-far problem and the multiple access interference. One way to circumvent these difficulties is to utilize a multiuser detector.

There are many optimum multiuser detectors for a DS-CDMA system, but they practically require a high computational complexity which grows exponentially with the number of users. To reduce the computational complexity, linear detectors may be employed because their complexity increase linearly with the number of users. The MMSE linear detector is one of such linear detectors.

The single-user matched filter detector is optimized to combat the background white noise, while the decorrelating detector eliminates the multiuser interference regardless of the background noise. Therefore, MMSE linear detector can be seen as a compromise solution that takes into account the background noise and the interference of each user.

The MMSE linear detector has shown that it approaches the conventional detector as $\sigma \to \infty$ and converges to the decorrelating detector as the signal-to-noise ratios go to infinity. As a result, the MMSE linear detector has the same asymptotic efficiency and near-far resistance as the decorrelating detector. Furthermore, the MMSE linear detector achieves optimum near-far resistance.

The decision algorithm for the MMSE detector can be implemented into two steps. First a linear estimate $\hat{\mathbf{b}} = \mathbf{My}$ of the transmitted bits $\mathbf{b}$ is obtained from the matched filter output $\mathbf{y}$. Second the estimate $\hat{\mathbf{b}}$ is compared to a threshold of zero, and the final bit estimates are obtained to be the value of sgn($\hat{\mathbf{b}}$). The key advantage of the MMSE detector is the ease with which it can be implemented in an adaptive mode, which requires no priori knowledge of signature waveforms, or in a blind mode, which avoids the need for training sequences altogether.

7. References

[1] T. Ottosson, "*Coding, modulation and multi-user decoding for DS/CDMA systems*," Ph.D. thesis, School of Electrical and Computer Engineering, Chalmers University of Technology, Gothenburg, Sweden, December 1997.

[2] Sergio Verdu, "*Multiuser Detection*," Cambridge University Press, New York, USA, 1998.

[3] A. Duel-Hallen, J.Holtzman, and Z. Zvonar, "*Multi-user detection for CDMA systems*," IEEE Personal Communications, vol.2, no.2, pp.46-58, April 1995.

[4] R. Lupas and S. Verdu, "*Near-far resistance of multi-user detectors in asynchronous channels*," IEEE Transactions on Communications, vol.38, no.4, pp.496-508, April 1990.

[5] Zhenhua Xie, Robert T. Short, and Craig K. Rushforth, "A Family of Subobtimum Detectors for Coherent Multiuser Communications," IEEE journal on selected areas in communications, Vol.8, No.4, May 1990.

[6] David Gesbert, Joakim Sorelius, Petre Stoica, and A. Paulraj, "*Blind Multiuser MMSE Detector for CDMA Signals in ISI Channels*," IEEE Communications Letters, Vol. 3, No. 8, August 1999.

[7] H. V. Poor and S. Verdu, "*Probability of error in MMSE multiuser detection*," IEEE Trans. Information Theory, 43:858-871, May 1997.

[8] Gordon L. Stuber, "Principles of Mobile Communication," Kluwer Academic Publisher, Massachusetts, 1999.

[9] R. Lupas and S. Verdu, "Linear multi-user detection for synchronous code-division multiple-access channels," IEEE Transactions on Information Theory, vol.35, no.1, pp.123-136, January 1989.

[10] P.D. Alexandar, L.K. Rasmussen, and C.B. Schlegel, "A linear receiver for coded multi-user CDMA," IEEE Transactions on Communications, vol.45, no.5, pp.605-610, May 1997.

[11] U. Fawer and B. Aazhang, "A multi-user receiver for code division multiple access communications over multipath channels," IEEE Transactions on Communications, vol.43, no.2/3/4, pp.1556-1565, Feb./Mar./Apr. 1995.