An MMSE Infinite Impulse Response Equalizer for Perpendicular Recording Channels with Jitter Noise

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Abstract: A finite impulse response (FIR) equalizer is practically employed in conjunction with the Viterbi detector for data detection process in magnetic recording channels. However, the FIR equalizer with a large number of taps is required at high density storage channels. It is well-known that an infinite impulse response (IIR) filter with a small number of taps can closely approximate such an FIR filter. In this paper, we propose the IIR filter for perpendicular recording channels, based on a minimum mean-squared error approach, and compare its performance with the FIR equalizer in the presence and in the absence of media jitter noise. Results indicate that the IIR equalizer performs better than the FIR equalizer for all jitter noise levels, especially when the number of equalizer taps is small (e.g., 3 taps) and the normalized recording density is high.

1. Introduction

The intersymbol interference (ISI) is a major disturbance in perpendicular magnetic recording channels, especially at high data storage densities. To cope with ISI, the techinique called partial-response maximum-likelihood (PRML) [1] is used for data detection process in magnetic recording channels. Practically, this technique employs a finite impulse response (FIR) equalizer to shape the read-back signal to a predetermined target [2] – [3] before performing maximum-likelihood (ML) equalization by the Viterbi detector [4].

In general, the FIR equalizer with a large number of taps is required to properly function at high density storage channels. However, the total number of equalizer taps is basically limited by the maximum allowable loop delay in the timing recovery loop [2] because a small loop delay provides more robust phase locking, which in turn improves the overall system performance. It has been known in the literature that an infinite impulse response (IIR) filter with a small number of taps can closely approximate the FIR filter.

The partial response (PR) targets of the form $(1+D)^n$ are suitable for perpendicular recording channels [5], where D is a delay operator and n is an integer. Given the PR target, its corresponding FIR equalizer can be obtained, based on the minimum mean-squared error (MMSE) approach [6]. This paper proposes the IIR equalizer for PR channels based also on the MMSE approach. Although the IIR filter has much concern about stability, based on extensive simulations, we have been able to conclude that the proposed IIR equalizer is highly stable for PR channels.

Several works related to the IIR equalizers have been studied and analyzed in the literature. For instance, reference [7] investigated the performance of using continuous-time adaptive IIR equalizers for EPR4 channels, and the IIR modeling was considered in the design of decision feedback equalizers to reduce the number of filter taps [8]. Reference [9] approximated a high density storage channel with a digital IIR filter so that the detector could incorporate this knowledge to improve the performance of noise-predictive maximum-likelihood (NPML) detection. Furthermore, reference [10] proposed the IIR equalizer for PR channels by minimizing the *filtered* error sequence, whose performance is better than the FIR equalizer when the number of equalizer taps is small. Nonetheless, in this paper, we design the IIR equalizer based on minimizing the *actual* error sequence. Then, we compare its performance with the IIR equalizer proposed in [10] and the FIR equalizer in the presence and in the absence of media jitter noise.

The rest of this paper is organized as follows. After explaining our system model in Section 2, we describe the design of the MMSE IIR equalizer for PR channels in Section 3. Simulation results are given in Section 4. Finally, Section 5 concludes this paper.

2. System Model

Consider the system model shown in Fig. 1, where a binary input sequence $x_k \in \{\pm 1\}$ with bit period T is filtered by an ideal differentiator $(1-D)/2$ to form a transition sequence $c_k \in \{-1, 0, 1\}$, where $c_k = \pm 1$ corresponds to a positive or a negative transition, and $c_k = 0$ corresponds to the absence of a transition. The transition sequence c_k passes through the magnetic recording channel represented by $g(t)$. The transition response $g(t)$ for perpendicular recording is [11]

$$g(t) = \operatorname{erf}\left(\frac{2t\sqrt{\ln 2}}{PW_{50}}\right), \tag{1}$$

where $\operatorname{erf}(y) = \frac{2}{\sqrt{\pi}}\int_0^y e^{-z^2}dz$ is an error function, and PW_{50} determines the width of the derivative of $g(t)$ at half its maximum.

In the context of magnetic recording, a *normalized recording density* is defined as ND = PW_{50}/T, which determines how many data bits can be packed within the resolution unit PW_{50}. The media jitter noise, Δt_k, is modeled as a random shift in the *transition position* with a Gaussian probability distribution function with zero mean and variance $|c_k|\sigma_j^2$ [5] (i.e.,

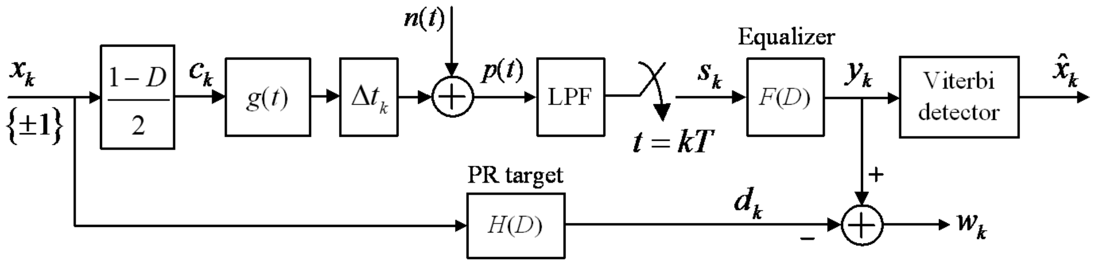

Figure 1. A system model with equalizer design.

$\Delta t_k \sim \mathcal{N}(0, |c_k|\sigma_j^2))$ truncated to $T/2$, where $|a|$ takes the absolute value of a, and σ_j is specified as a percentage of T. Clearly, the severity of media jitter noise depends on how large the value of σ_j is.

The read-back signal, $p(t)$, can then be expressed as [12]

$$p(t) = \sum_{k=-\infty}^{\infty} c_k g(t - kT + \Delta t_k) + n(t), \tag{2}$$

where $n(t)$ is additive white Gaussian noise with two-sided power spectral density $N_0/2$. The read-back signal $p(t)$ is filtered by a seventh-order Butterworth low-pass filter (LPF) and is then sampled at time $t = kT$, assuming perfect synchronization. The sampler output s_k is equalized by an equalizer, $F(D)$, such that the output sequence, y_k, resembles the desired sequence, d_k. Eventually, the Viterbi detector performs sequence detection to determine the most likely input sequence.

3. Design of an MMSE IIR Equalizer

In practice, a read-channel chip utilizes an FIR equalizer of the form

$$F_{\mathrm{FIR}}(D) = \sum_{k=-K}^{K} f_k D^k, \tag{3}$$

where K is an integer and f_k is the k-th coefficient of $F_{\mathrm{FIR}}(D)$, to shape the read-back signal to a predetermined target before performing ML equalization by the Viterbi detector. The design of the target $H(D) = \sum_{k=0}^{\nu} h_k D^k$, where ν is the target memory, and its corresponding FIR equalizer based on the MMSE approach is given in [6]. In this paper, we propose the design of the MMSE IIR equalizer for a given PR target.

Consider a block diagram for designing the MMSE IIR equalizer shown in Fig. 2. For simplicity, we consider the IIR equalizer of the form

$$F(D) = \frac{B(D)}{A(D)} = \frac{\sum_{k=-N}^{N} b_k D^k}{\sum_{k=0}^{M} a_k D^k}, \tag{4}$$

where N and M are integers, and b_k and a_k are the k-th coefficient of the numerator and the denominator of $F(D)$, respectively. For a given PR target, the aim is to find the suitable coefficients, a_k's and b_k's, such that the resulting IIR equalizer performs better than the FIR equalizer, when the number

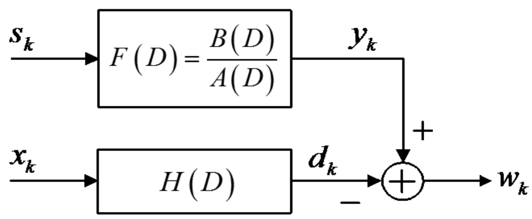

Figure 2. A block diagram for designing the IIR equalizer.

of equalizer taps is small and the ND is high. This can be achieved by designing the IIR equalizer such that an error sequence, w_k, in Fig. 2 is minimized.

Given the PR target $H(D)$ and the input data sequence x_k, it can be implied that the data sequence $d_k = x_k * h_k$ is known. Therefore, from Fig. 2, it is apparent that

$$y_k = s_k * f_k, \tag{5}$$
$$y_k * a_k = s_k * b_k, \tag{6}$$

where s_k and y_k is the input and the output of the IIR equalizer $F(D)$, respectively. Because $y_k = d_k + w_k$, substituting y_k into (6) gives

$$(d_k + w_k) * a_k = s_k * b_k, \tag{7}$$
$$w_k * a_k = (s_k * b_k) - (d_k * a_k), \tag{8}$$

It should be noted that the IIR equalizer proposed in [10] was designed to minimizing the *filtered* error sequence $v_k = w_k * a_k$ in (8). Nevertheless, in this paper, we directly minimize the *actual* error sequence w_k. We consider the case where $a_0 = 1$. Thus, (8) can be rewritten as

$$w_k = \mathbf{s}^{\mathrm{T}}\mathbf{b} - \tilde{\mathbf{d}}^{\mathrm{T}}\tilde{\mathbf{a}} - \tilde{\mathbf{w}}^{\mathrm{T}}\tilde{\mathbf{a}} - d_k, \tag{9}$$

where

$$\mathbf{s} = [s_{k+N}, \ldots, s_k, \ldots, s_{k-N}]^{\mathrm{T}}, \tag{10}$$
$$\mathbf{b} = [b_{-N}, \ldots, b_0, \ldots, b_N]^{\mathrm{T}}, \tag{11}$$
$$\tilde{\mathbf{d}} = [d_{k-1}, d_{k-2}, \ldots, d_{k-M}]^{\mathrm{T}}, \tag{12}$$
$$\tilde{\mathbf{a}} = [a_1, a_2, \ldots, a_M]^{\mathrm{T}}, \tag{13}$$
$$\tilde{\mathbf{w}} = [w_{k-1}, w_{k-2}, \ldots, w_{k-M}]^{\mathrm{T}}, \tag{14}$$

are $(2N+1)$–, $(2N+1)$–, M–, M–, and M–element column vectors, respectively, and $[\cdot]^{\mathrm{T}}$ is the transpose operation.

The MMSE IIR equalizer $F(D)$ can then be obtained by minimizing the mean-squared error of (9), i.e., $E\{w_k^2\}$, with respect to $\tilde{\mathbf{a}}$ and $\mathbf{b}$. This minimization process yields

$$\underbrace{\begin{bmatrix} \mathbf{R} & -\mathbf{X} \\ -\mathbf{X}^{\mathrm{T}} & \mathbf{D}+\mathbf{V}+\mathbf{V}^{\mathrm{T}}+\mathbf{W} \end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix} \mathbf{b} \\ \tilde{\mathbf{a}} \end{bmatrix}}_{\mathbf{z}} = \underbrace{\begin{bmatrix} \mathbf{c} \\ -\mathbf{q} \end{bmatrix}}_{\mathbf{y}}, \quad (15)$$

where

$$\begin{aligned} \mathbf{R} &= E\{\mathbf{s}\mathbf{s}^{\mathrm{T}}\}, & (16)\\ \mathbf{X} &= E\{\mathbf{s}\tilde{\mathbf{d}}^{\mathrm{T}}\} + E\{\mathbf{s}\tilde{\mathbf{w}}^{\mathrm{T}}\}, & (17)\\ \mathbf{D} &= E\{\tilde{\mathbf{d}}\tilde{\mathbf{d}}^{\mathrm{T}}\}, & (18)\\ \mathbf{V} &= E\{\tilde{\mathbf{d}}\tilde{\mathbf{w}}^{\mathrm{T}}\}, & (19)\\ \mathbf{W} &= E\{\tilde{\mathbf{w}}\tilde{\mathbf{w}}^{\mathrm{T}}\}, & (20) \end{aligned}$$

are $(2N+1)$-by-$(2N+1)$, $(2N+1)$-by-M, M-by-M, M-by-M, and M-by-M matrices, respectively, and

$$\begin{aligned} \mathbf{c} &= E\{\mathbf{s}d_k\}, & (21)\\ \mathbf{q} &= E\{\tilde{\mathbf{d}}d_k\} + E\{\tilde{\mathbf{w}}d_k\}, & (22) \end{aligned}$$

are $(2N+1)$– and M–element column vectors, respectively.

Because the matrix $\mathbf{A}$ in (15) is a square matrix, the coefficients of $F(D)$, i.e., $\mathbf{z}$, can then be easily obtained by

$$\mathbf{z} = \mathbf{A}^{-1}\mathbf{y}. \quad (23)$$

Additionally, from extensive simulations, we have been able to conclude that the proposed IIR equalizer is highly stable for PR channels in perpendicular recording systems.

4. Simulation Results

We consider the PR target $H(D) = 1+4D+6D^2+4D^3+D^4$ for perpendicular recording. The $(2K+1)$-tap FIR equalizer is designed based on the MMSE approach [6], which also yields an error sequence w_k that will be used to design our proposed IIR equalizer. The signal-to-noise ratio (SNR) is defined as

$$\mathrm{SNR} = 10\log_{10}\left(\frac{E_i}{N_0}\right), \quad (24)$$

in decibel (dB), where E_i is the energy of the channel impulse response. All equalizers are designed at SNR required to achieve bit-error rate (BER) of 10^{-5}, and each BER point is computed using as many 4096-bit data sectors as needed to collect 500 error bits, whereas only one data sector is used to design the proposed IIR equalizer. Moreover, we denote the IIR equalizer proposed in [10] as "M1," and the MMSE IIR equalizer proposed in this paper as "M2."

Figure 3(a) compares the performance of different equalizers as a function of NDs in the absence of jitter noise (i.e., $\sigma_j = 0\%$), where "Mx-vZmP" denotes the M1 or the M2 equalizer (i.e., $x \in \{1,2\}$) with $v = 2N$ zeros (equivalent to $v+1$ taps) and $m = M$ poles. As illustrated in Fig. 3(a), when the number of equalizer taps is small (e.g., 3 taps) and ND is high, both the M1 and the M2 equalizers perform better than the FIR equalizer, and the M2 equalizer is slightly better than the M1 equalizer. This is because the IIR equalizer

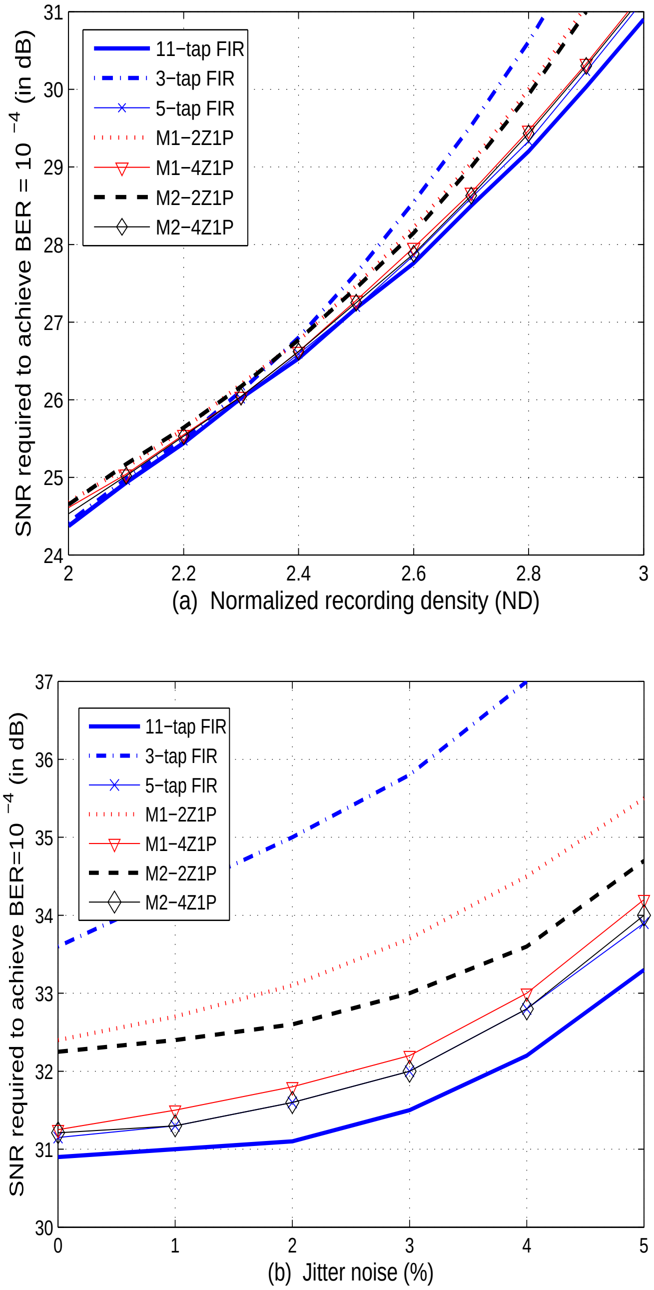

Figure 3. Performance comparison.

can shape the read-back signal to the PR target better than the FIR equalizer. In Fig. 3(b), we pick ND = 3, and this time compare the performance of different equalizers as a function of the jitter noise amount. Again, when the number of equalizer taps is small (i.e., 3 taps), it is clear that both the M1 and the M2 equalizers require lower SNR to achieve BER = 10^{-4} than the FIR equalizer for all jitter noise amounts, and the M2 equalizer performs better than the M1 equalizer.

The reason that the IIR equalizers provide better performance than the FIR equalizer might be because they can shape the read-back signal to the PR target better than the FIR equalizer does, especially when the number of equalizer

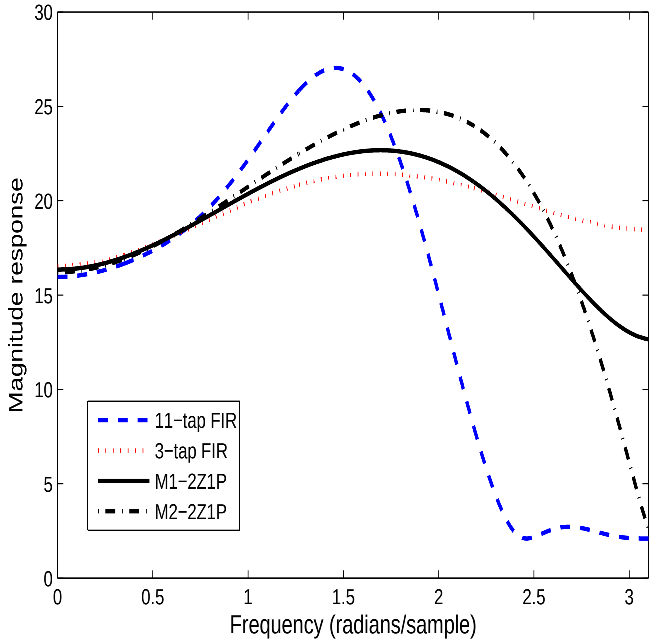

Figure 4. Frequency responses of equalizers at ND = 3.

taps is small. This can be explained by plotting the frequency responses of different equalizers in Fig. 4 for perpendicular recording channels at ND = 3. If we assume that the 11-tap FIR equalizer is the best, the 3-tap equalizer whose frequency response closely matches the frequency response of the 11-tap FIR equalizer will yield the best performance among the 3-tap equalizers. Clearly, the M1 and the M2 equalizers give a better match to the frequency response of the 11-tap FIR equalizer than the 3-tap FIR equalizer. Furthermore, the M2 equalizer seems to provide a better match to the frequency response of the 11-tap FIR equalizer than the M1 equalizer. This explains why the M2 equalizer performs better than the M1 equalizer.

5. Conclusion

This paper proposed the IIR equalizer, denoted as "M2," for PR channels in perpendicular recording systems, based on the MMSE approach. Unlike the IIR equalizer proposed in [10], denoted as "M1," which was designed to minimize the *filtered* error sequence, our proposed IIR equalizer is designed by minimizing the *actual* error sequence. Nevertheless, both the M1 and the M2 equalizers require the knowledge of the PR target.

Simulation results have indicated that, when the number of equalizer taps is small (e.g., 3 taps) and ND is high, the IIR equalizers (M1 and M2) perform better than the FIR equalizer, designed based on the MMSE approach [6], for all jitter noise levels. In addition, it is clear that the M2 equalizer also provides better performance than the M1 equalizer all jitter noise levels. Although the IIR filter has some concern about stability, we found that the proposed IIR equalizer is highly stable for PR channels.

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