# ROBUSTNESS OF PER-SURVIVOR ITERATIVE TIMING RECOVERY AGAINST PATTERN-DEPENDENT NOISE IN PERPENDICULAR RECORDING CHANNELS

Piya Kovintavewat

Faculty of Science and Technology, Nakhon Pathom Rajabhat University 85 Malaiman rd., Muang district, Nakhon Pathom 73000, Thailand piya@npru.ac.th

#### ABSTRACT

Per-survivor iterative timing recovery was proposed by Kovintavewat, Barry, Erden, and Kurtas [1] to perform timing recovery, equalization, and error-correction decoding jointly. In this paper, we investigate the robustness of this scheme in the situation where the channel is dominated by media jitter noise. We apply the pattern-dependent noise-predictive (PDNP) technique to per-survivor iterative timing recovery. Results indicate that the per-survivor iterative timing recovery with the PDNP technique is more robust than other iterative timing recovery schemes when operating in media noise dominated channel.

**Keywords:** pattern-dependent noise-predictive, persurvivor iterative timing recovery, magnetic recording.

## 1. INTRODUCTION

Timing recovery is a crucial component in magnetic recording systems. It is the process of synchronizing the sampler with the received analog signal. The quality of synchronization has a devastating impact on overall performance.

Per-survivor iterative timing recovery was proposed in [2] to deal with the problem of timing recovery operating at low signal-to-noise ratio (SNR). It is realized by first developing a per-survivor soft-output Viterbi algorithm (SOVA) [3], denoted as "PSP-SOVA," [2] by embedding the timing recovery step inside the SOVA equalizer using per-survivor processing (PSP) [4], a technique of jointly estimating a data sequence and unknown parameters. Then, the proposed scheme iteratively exchanges soft information between PSP-SOVA and a soft-in soft-out (SISO) decoder. As studied in [2], the proposed scheme outperforms the conventional receiver, especially when the timing error is severe.

In practice, noise in magnetic recording channels is also data-dependent [5], whose severity depends on the data pattern written on the disk. Media jitter noise can be given as an example of this noise. A patterndependent noise-predictive (PDNP) technique has been proposed in [5] to combat with data-dependent noise. In this paper, we apply the PDNP technique in PSP-SOVA, resulting in "PSP-SOVA-PDNP." Then, we investigate the robustness of per-survivor iterative timing recovery in data-dependent noise dominated channels.

This paper is organized as follows. After describing our channel model in Section 2, we summarize how pattern-dependent noise predictor performs in Section 3. Section 4 explains the complexity of PSP-SOVA-PDNP. Simulation results are given in Section 5. Finally, Section 6 concludes this paper.

## 2. CHANNEL DESCRIPTION

Consider the coded channel model shown in Fig. 1, where a message sequence  $x_k \in \{0,1\}$  is encoded by an error-correction encoder and is mapped to a coded sequence  $a_k \in \{\pm 1\}$ . The coded sequence  $a_k$  with bit period T is filtered by (1-D)/2, where D is the delay operator, to form a transition sequence  $b_k \in \{-1, 0, 1\}$ , where  $b_k = \pm 1$  corresponds to a positive or a negative transition, and  $b_k = 0$  corresponds to the absence of a transition. The transition sequence  $b_k$  passes through a perpendicular recording channel whose transition response is given by  $g(t) = \operatorname{erf}(2t\sqrt{\ln 2}/\mathrm{PW}_{50})$  [6], where  $\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-z^2} dz$  is an error function, and  $\mathrm{PW}_{50}$ determines the width of the derivative of q(t) at half its maximum. In the context of magnetic recording, a normalized recording density is defined as  $ND = PW_{50}/T$ , which determines how many data bits can be packed within the resolution unit  $PW_{50}$ .

The read-back signal, p(t), can then be expressed as [1]

$$p(t) = \sum_{k} (a_k/2) \{ g(t - kT - \Delta t_k - \tau_k) - g(t - (k+1)T - \Delta t_{k+1} - \tau_k) \} + n(t) (1)$$

where n(t) is additive white Gaussian noise with two-

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Figure 1: Channel model with target design.

sided power spectral density  $N_0/2$ . The media jitter noise,  $\Delta t_k$ , is modeled as a random shift in the *transition position* with a Gaussian probability distribution function with zero mean and variance  $|b_k|\sigma_j^2$  (i.e.,  $\Delta t_k \sim \mathcal{N}(0, |b_k|\sigma_j^2)$ ) truncated to T/2, where |x| takes the absolute value of x. The clock jitter noise,  $\tau_k$ , is modeled as a random walk [7] according to  $\tau_{k+1} =$  $\tau_k + \mathcal{N}(0, \sigma_w^2)$ , where  $\sigma_w$  determines the severity of the timing jitter. The random walk is chosen because its simplicity and its ability to represent a variety of channels by changing only one parameter.

At the receiver, the read-back signal p(t) is filtered by a seventh-order Butterworth low-pass filter (LPF) and is sampled at time  $kT + \hat{\tau}_k$ , where  $\hat{\tau}_k$  is a receiver's estimate of  $\tau_k$ . The sampler output  $s_k$  is equalized by an equalizer, F(D), such that an equalizer output,  $y_k$ , closely resembles a desired sample,  $r_k$ . Note that the design of a target and its corresponding equalizer can be found in [8]. Conventional timing recovery is based on a second-order phase-locked-loop (PLL), which updates the sampling phase offset according to [9, 10]

$$\hat{\theta}_{k+1} = \hat{\theta}_k + \beta \{ y_k \hat{r}_{k-1} - y_{k-1} \hat{r}_k \}, \qquad (2)$$

$$\hat{\tau}_{k+1} = \hat{\tau}_k + \alpha \{ y_k \hat{r}_{k-1} - y_{k-1} \hat{r}_k \} + \hat{\theta}_{k+1}, \quad (3)$$

where  $\hat{r}_k$  is an estimate of  $r_k$ ,  $\hat{\theta}_k$  represents an estimate of frequency error, and  $\alpha$  and  $\beta$  are the PLL gain parameters.

In the conventional receiver, conventional timing recovery is followed by a turbo equalizer [11] (see Fig. 1), which iteratively exchanges soft information between the SISO detector and the SISO decoder.

## 3. PATTERN-DEPENDENT NOISE PREDICTION

The key idea of the PDNP technique is to include the noise predictor in the branch metric calculation of the SISO detection algorithm. Specifically, the branch metric at time k for the transition from state u to state v,

 $\rho_k(u, v)$ , can be written as [5]

$$\rho_k(u,v) = \log(\sigma_p(u,v)) + \frac{|y_k - \hat{r}_k(u,v) - \hat{n}_k(u,v)|^2}{2\sigma_p^2(u,v)},$$
(4)

where  $\hat{n}_k(u, v) = \sum_{i=1}^{L} p_i(u, v) \{y_{k-i} - r_{k-i}(u, v)\}$  is the predicted noise sample associated with the data pattern that corresponds to (u, v), L is the predictor order,  $p_i(u, v)$  is the *i*-th noise predictor coefficient associated with (u, v), and  $\sigma_p^2(u, v)$  is the prediction error variance associated with (u, v).

Note that  $p_i(u, v)$  and  $\sigma_p(u, v)$  can be found by solving normal equations as described in [5], and we then use the same coefficients for all data sectors even if the noise characteristics change from sector to sector. Since the values of  $p_i(u, v)$  and  $\sigma_p(u, v)$  depend on the data pattern associated with (u, v), this PDNP technique requires the number of trellis [12] states of  $Q_p = 2^{\nu+L}$ , where  $\nu$  is target memory.

#### 4. COMPLEXITY ANALYSIS OF PSP-SOVA-PDNP

PSP-SOVA-PDNP works in a same manner as PSP-SOVA does [2], except that we replace the branch metric calculation in (A-6) [2] with (4), and keep track of the predicted noise  $\hat{n}_k(u, v)$  of each transition. Clearly, PSP-SOVA-PDNP has high complexity because it requires trellis expansion. To reduce its complexity, we compute the predicted noise  $\hat{n}_k(u, v)$  based on tentative decisions associated with each survivor path, thus requiring no trellis expansion. We call this *modified* PSP-SOVA-PDNP as "PSP-SOVA-PDNP-MO."

To help quantify how much computational complexity PSP-SOVA-PDNP-MO contains when compared with PSP-SOVA-PDNP, we measure *computational complexity*<sup>1</sup> by counting *only* the total number of additions and multiplications (*per bit*). For other mathematical

 $<sup>^1 \</sup>mathrm{In}$  this paper, we ignore the complexity of hardware implementation.

functions, e.g.,  $\log(x)$ ,  $\exp(x)$ , etc., we assume they are implemented as lookup tables, and that we ignore their complexity. It can be shown that PSP-SOVA-PDNP has  $(4N_s + N_e + 4L + 7)Q_p + \frac{\delta^2 + 9\delta + 9}{2} + 1$  additions and  $(N_s + N_e + 2L + 11)Q_p + 1$  multiplications, whereas PSP-SOVA-PDNP-MO has  $(4N_s + N_e + 2L + 8)Q + \frac{\delta^2 + 9\delta + 9}{2} + 1$  additions and  $(N_s + N_e + 2L + 11)Q + 1$  multiplications, where  $\delta = 5(\nu + 1)$  is a SOVA decoding depth [3],  $Q = 2^{\nu}$  is the number of trellis states,  $N_s$  is the number of interpolation taps used to refine the samples [2], and  $N_e$  is the number of equalizer taps. It is apparent that PSP-SOVA-PDNP-MO has lower complexity than PSP-SOVA-PDNP.

#### 5. SIMULATION RESULTS

We consider a rate-8/9 coded system in which a block of 3640 message bits,  $\{x_k\}$ , is encoded by a regular (3, 27) LDPC code [13], resulting in a coded block length of 4095 bits,  $\{a_k\}$ . The SISO detector is implemented based on SOVA, and the SISO decoder is implemented based on the message passing algorithm with 5 internal iterations (i.e.,  $N_i = 5$ ). To account for a coded system, we define a user density,  $D_u$ , as  $D_u = ND/code$  rate. Also, we consider a perpendicular recording channel in a moderate condition, e.g., with  $\sigma_w/T = 0.5\%$  clock jitter noise and 0.2% frequency offset. The SNR is defined as SNR =  $10 \log 10(E_i/N_0)$  in dB, where  $E_i$  is the energy of the channel impulse response (the derivative of the transition response scaled by 2). The generalized partial response (GPR) target [14] and a 21-tap (i.e.,  $N_e = 21$ ) equalizer are designed at SNR required to achieve BER =  $10^{-5}$ .

The robustness of per-survivor iterative timing recovery in high media noise environment is investigated. We focus on comparing the performance of different schemes when they have same complexity. To do so, we count the number of operations (per bit) of different schemes, including an LDPC decoder. Note that it can be shown the LDPC decoder requires  $(j + (k - 1)(1 - R))N_i + 1$  additions and  $(1 - R)N_i$  multiplications, where (j,k) = (3,27) is an LPDC parameter, and R = 1 - j/k is a code rate.

Let N be the number of iterations. Thus, by using  $\nu = 3$ , L = 3,  $N_s = 21$ ,  $N_e = 21$ ,  $N_i = 5$ , we can show that the conventional receiver (performing PDNP in SOVA) has 70 + 1541.9N additions and 41 + 897.56N multiplications; per-survivor iterative timing recovery using PSP-SOVA-PDNP has 8261.9N additions and 3777.6N multiplications; and per-survivor iterative timing recovery using PSP-SOVA-PDNP-MO has 1277.9N additions and 473.56N multiplications. It should be pointed out that multiplication has much more complexity than addition in terms of circuit implementation. Hence, we consider only the number of multiplications when comparing the performance of dif-



Figure 2: Complexity comparison.

ferent schemes. Fig. 2 compares the number of multiplications of each scheme. Clearly, per-survivor iterative timing recovery using PSP-SOVA-PDNP has very high complexity if compared to the others.

In addition, we also assume that current technology can support the total number of multiplications equal to 1 iteration of per-survivor iterative timing recovery using PSP-SOVA-PDNP, which is approximately equal to 8 iterations of per-survivor iterative timing recovery using PSP-SOVA-PDNP-MO, and 4 iterations of the conventional receiver (see Fig. 2). Fig. 3 compares the performance of different schemes when they have same complexity for  $D_u = 3$ ,  $\sigma_j/T = 10\%$ , and a 4-tap GPR target. Apparently, for low to moderate complexity, per-survivor iterative timing recovery using PSP-SOVA-PDNP-MO performs better than other schemes even if it yields worse performance when the number of iterations is fixed instead of system complexity.

### 6. CONCLUSION

Per-survivor iterative timing recovery jointly performs timing recovery, equalization, and error-correction decoding. We have shown that it is more robust against data-dependent noise than the conventional iterative architecture where timing recovery and iterative decoding are performed separately.

#### 7. REFERENCES

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Figure 3: Performance comparison with same complexity.

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